A crash module on the Physics of Nuclei

- Aims: Provide necessary background to move on, and Illustrate how the ideas developed in atoms & molecules, such as why things bind, Independent Particle Approximation, Pauli Exclusion Principle, Spin-orbit interaction, ... work in the Physics of Nuclei

Background on nuclear physics

(i) Why do nucleons bind to form a nucleus?

- Nucleon: collective term for proton and neutron

  proton: $938.27\ \frac{MeV}{c^2}$ * ; neutron: $939.57\ \frac{MeV}{c^2}$ *

  [quite similar] [except +e charge of p]

- Forming a nucleus is more stable than isolated nucleons

  [same reason for why there are atoms & molecules]

- Binding energy quantifies nucleus stability

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* Using $1u = 1.66054\times10^{-27}\ Kg = 931.49\ MeV/c^2 = \frac{1}{12}\cdot$ mass of ${}^{12}_{6}C$ atom,

$m_{proton} = 1.007276\,u$, $m_{neutron} = 1.008665\,u$, $m_{electron} = 0.511\ MeV/c^2 = 5.486\times10^{-4}\,u$.

- Mass - Energy (mc^2 relation)

Nucleus ${}^{A}_{Z}X_{N}$ $\quad Z = \underbrace{\#\text{ protons}}_{\text{which element}}$, A = mass number (# nucleons)

symbol of element

$A = Z + N$ ← # neutrons

Formally, $E_{binding} = \underbrace{\left(Z m_p + N m_n - m_{nucleus}\right)}_{\text{mass (energy) lowered in binding into nucleus}} c^2$

Practically, put electrons back

$$B \equiv E_{binding} = \Big[Z\underbrace{(m_p + m_e)} + N m_e - \underbrace{(m_{nucleus} + Z m_e)}\Big] c^2$$

binding energy

$$\approx \Big[Z\, m({}^{1}_{1}H) + N m_e - m({}^{A}_{Z}X)\Big] c^2 \quad \text{(convenient formula)}$$

atomic mass of ${}^{1}_{1}H$ $\quad$ atomic mass of ${}^{A}_{Z}X$

[data table]

Aside:

$$(Zm_p + Nm_n)c^2 = \text{Energy of isolated nucleons}$$

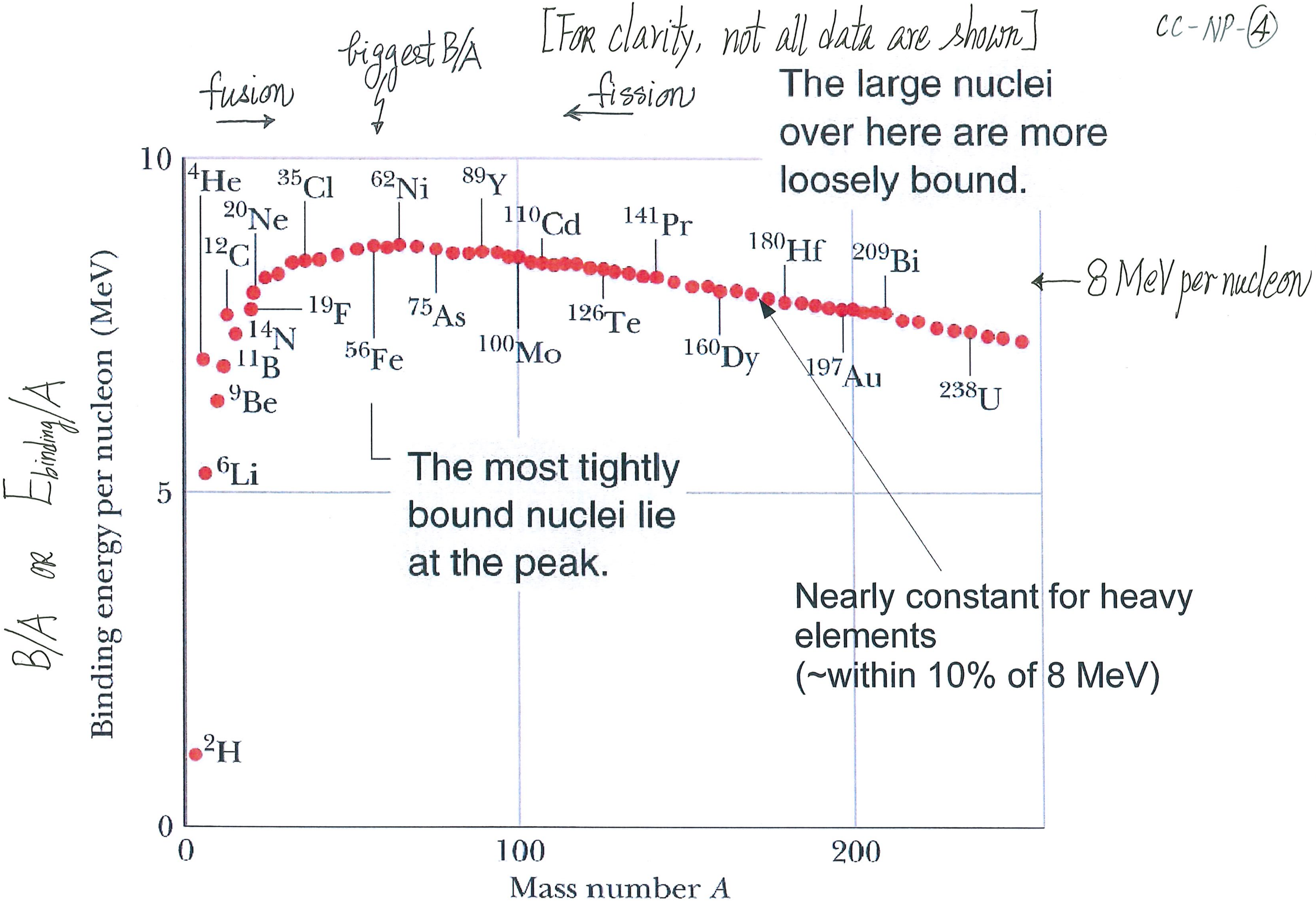
$$m_{nucleus}c^2 = \text{Energy of nucleus}$$

Fact that the nucleus is there $\Rightarrow m_{nucleus}c^2 < (Zm_p + Nm_n)c^2$

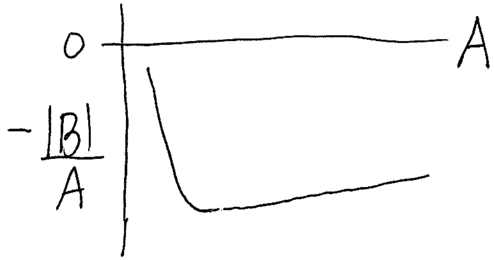
$$E_{binding} = \underbrace{(Zm_p + Nm_n - m_{nucleus})c^2}_{\text{quantifies stability of nucleus}} \quad (>0)$$

(bigger (positive) $E_{binding} \Rightarrow$ more stable)

c.f. 13.6 eV is the energy lowered when a proton and an electron bind to form a hydrogen atom (but $13.6\,eV \Leftrightarrow \underbrace{2.42\times10^{-35}\,kg \text{ OR } 1.46\times10^{-8}\,u}_{\text{tiny}}$)


[For clarity, not all data are shown]
fusion
biggest B/A
fission
The large nuclei over here are more loosely bound.
8 MeV per nucleon
B/A or E binding/A
Binding energy per nucleon (MeV)
Mass number A
10
5
0
100
200
4He
35Cl
62Ni
89Y
20Ne
110Cd
141Pr
12C
180Hf
209Bi
19F
75As
14N
126Te
11B
56Fe
100Mo
160Dy
197Au
238U
9Be
6Li
2H
The most tightly bound nuclei lie at the peak.
Nearly constant for heavy elements
(~within 10% of 8 MeV)

- In Nuclear Physics, B/A is plotted so that a higher B/A is more stable.
- If you insist on having a graph that the most stable nucleus corresponds to a minimum, then flip the curve upside down



y-axis: $\dfrac{(m_{nucleus} - Zm_p - Nm_N)c^2}{A}$ (flipped terms)

- Fusion

$2\,{}^2_1H \rightarrow {}^4_2He$ [climb up the B/A curve]

more stable

$\Rightarrow$ $\underbrace{\text{energy}}_{\sim 23\text{ MeV}}$ is released

[c.f. ~ eV energy in chemical processes]

Fission [climb up B/A curve]

${}^{235}_{92}U$ ~ $B/A \cong 8$ MeV

↳ split into smaller nuclei of $B/A \sim 8.8$ MeV

$\Rightarrow$ more stable $\Rightarrow$ energy is released

$235 \times \underbrace{(0.8)}_{\text{per nucleon}} \simeq 188$ MeV (released)

Fusion Energy

Why we are here!

Energy from the Sun

Fission Energy

What we are using, e.g.

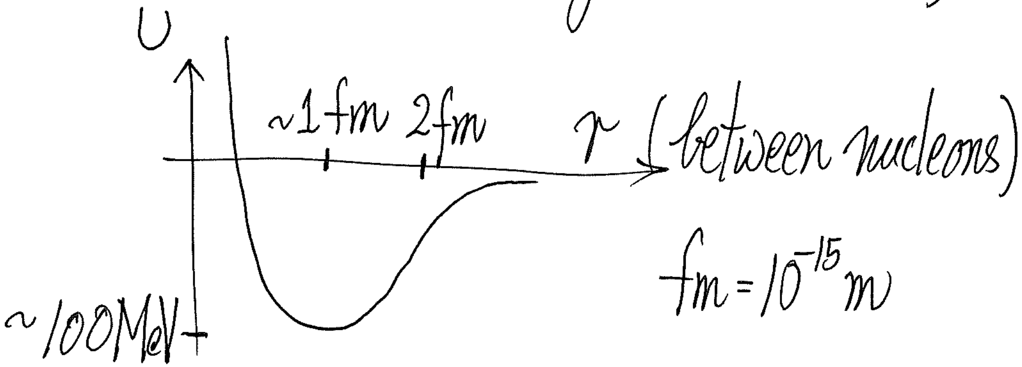
nuclear plants in Daya Bay

Life and living are supported by "climbing up the B/A curve"!

(ii) <u>How do nucleons bind?</u> ${}^2_1H$ exists $\Rightarrow$ p & n bind

Nuclear force between nucleons [n-n, n-p, p-p]

- Short-range ($\sim 10^{-15}$ m)
- strong [overcome Coulomb repulsion between protons in nuclei]



$\text{fm} = 10^{-15}\,\text{m}$

- spin-dependent [from deuteron, ${}^2_1H$ data] ($S = 1$) (p, n are spin-half particles)

No standard mathematical form.

<u>Yukawa (湯川秀樹) Potential energy Form</u> (1935)

$$U(r) \propto -\frac{g^2}{r} e^{-r/b} + (\text{repulsive part})$$ (1949 Nobel Physics Prize)

g^2 sets the strength, b (OR R) sets the range

- R (or b) is related to the mass of a <u>force mediator</u>

$$R = \frac{\hbar}{mc}$$ m = mass of particle that mediates the interaction

$$\boxed{m = \frac{\hbar}{cR}}$$

for $R \sim 10^{-15}$ m, $\underbrace{m \sim 200 \ \frac{\text{MeV}}{c^2}}$

Yukawa's <u>meson</u> (介子) [found in 1947]

- <u>Coulomb</u> Form for Comparison

$$U_{Coulomb}(r) = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} \quad \text{(SI units)}$$

$$= \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot \frac{\hbar c}{r} \quad [\hbar c = 197 \text{ MeV}\cdot\text{fm}]$$

$$= \underbrace{\alpha}_{1/137} \cdot \frac{\hbar c}{r}$$

$$= \alpha \cdot \frac{1}{r} \quad \text{(in units with } c=1, \hbar=1)$$

fine structure constant sets strength of EM interaction

<u>Yukawa:</u> $U(r) = -g^2 \frac{1}{r} e^{-r/b}$

g^2 sets the strength of interaction

Aside: Making Connection

- Earlier saw that for (an excited state in atom) something that lasts only an average time τ, there is an uncertainty in energy $\Delta E = \hbar/\tau$.

- Yukawa: Interaction can be viewed as mediated by exchanging a particle that only lasts (exists) for a short while

[Big Moment in Physics! Idea persists in modern particle physics!

exist for a time $\tau \sim \frac{R}{c}$ (c is the highest speed possible, just an estimate)

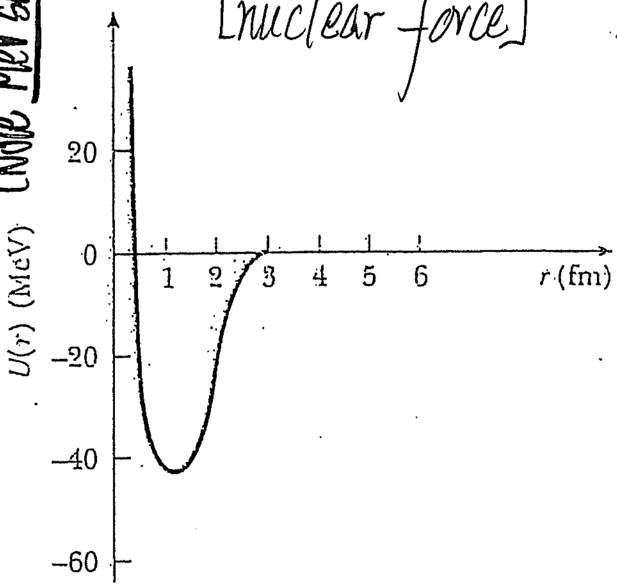
$$\frac{\Delta E}{c^2} = m = \frac{\hbar}{\tau} \cdot \frac{1}{c^2} = \frac{\hbar}{Rc}$$

mass of Yukawa's π-meson

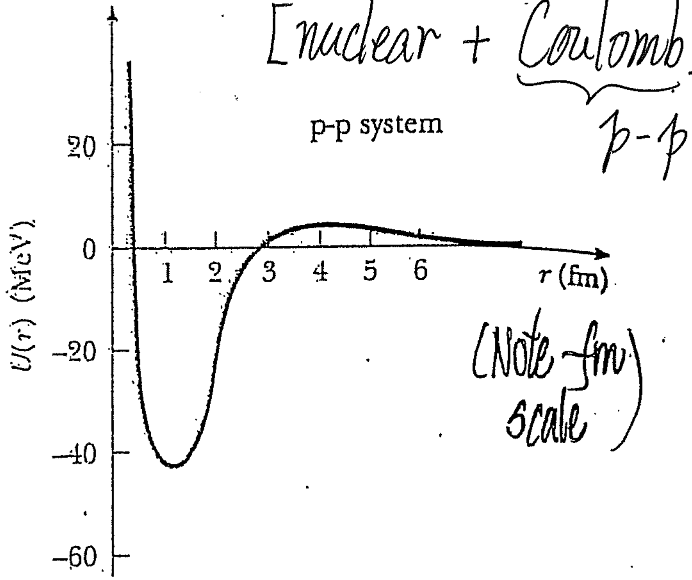
[range of force $\sim \frac{1}{\text{mass of force carrier}}$]

Schematically [Note MeV scale]

When n sees p (or n)
[nuclear force]



When p sees p
[nuclear + $\underbrace{\text{Coulomb}}_{p-p}$]

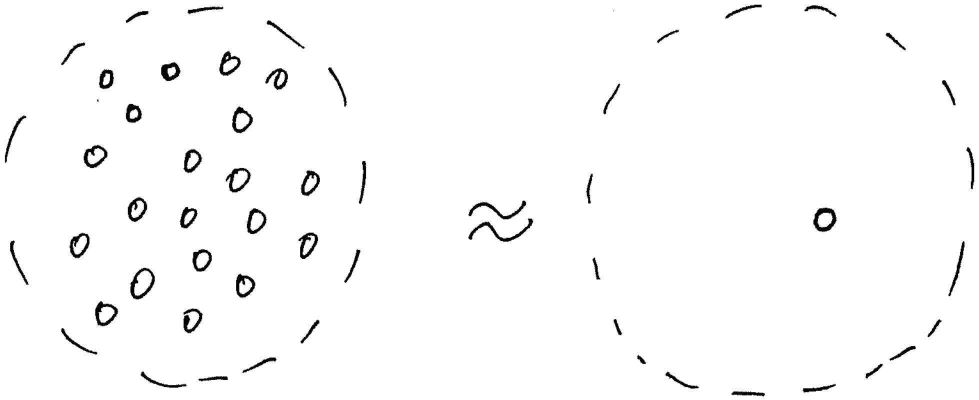


- Nucleus is a <u>many-nucleon problem</u> [c.f. <u>atom</u> is a many-electron problem]

• Avoid it!	• Avoid it!
• Independent particle approximation	• Independent particle approximation [behind it is Hartree Approximation]
• Single-proton states + Pauli Principle Single-neutron states + Pauli Principle	• single-electron states + Pauli Principle

(iii) Independent Particle Approximation (IPA)

- Many-proton & Many-neutron nuclei



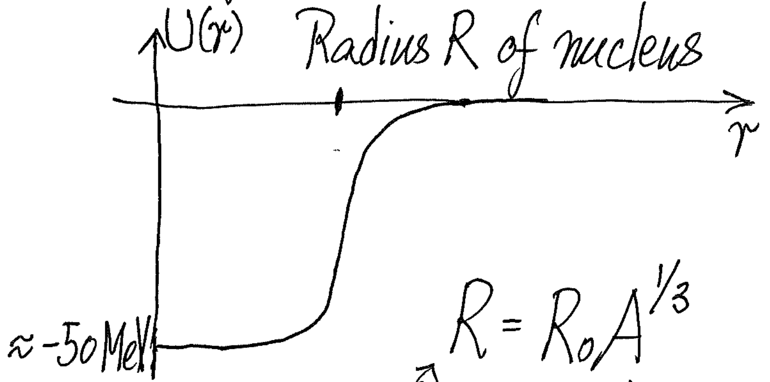
Many-nucleon

Single-nucleon problem

but in a $\underbrace{V_{eff}(\vec{r})}$

due to other nucleons on average (nuclear force) and Coulomb on average (if "o" is a proton)

- Due to nuclear force alone, each nucleon sees an averaged $U(r)$ of the form



$R = R_0 A^{1/3}$

radius of nucleus (R); # nucleons (A)

$R_0 \approx 1.2$ fm (Exp'tally)

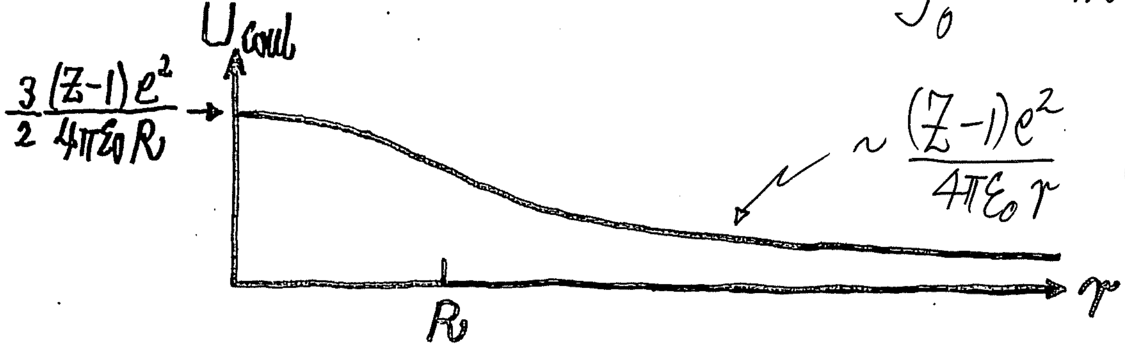
(3D spherically symmetrical!)
(at least approximately)

- But proton also experiences Coulomb energy due to $(Z-1)$ other protons

Model:  uniformly charged $+(Z-1)e$ sphere of radius R (EM problem)

$$\text{EM} \Rightarrow \vec{E}(r) = \begin{cases} \dfrac{\rho}{3\varepsilon_0}\, r\, \hat{r} & (r<R) \\[2ex] \dfrac{\rho}{3\varepsilon_0}\dfrac{R^3}{r^2}\hat{r} & (r>R) \end{cases}, \quad \rho = \frac{(Z-1)e}{\frac{4\pi}{3}R^3}$$

$$V(r) = -\int_0^r \vec{E}\cdot d\vec{l} \quad \text{and} \quad U_{Coul}(r) = eV(r)$$



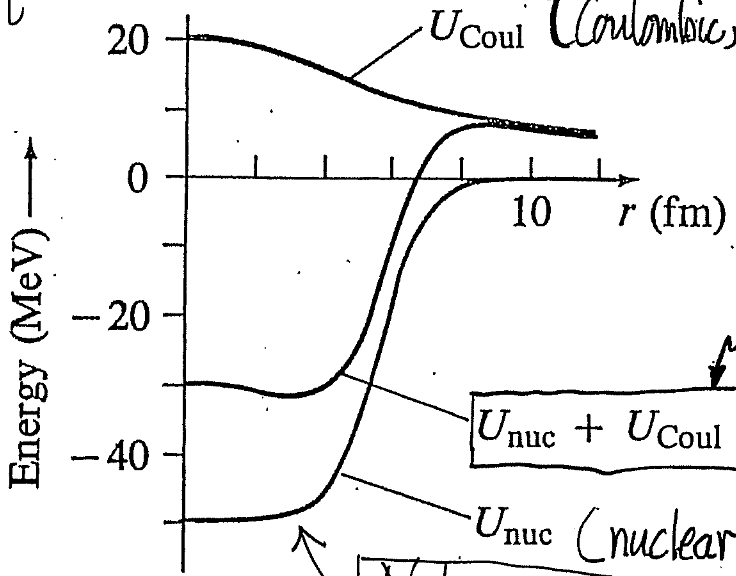
Typically, $U_{Coul} \sim$ few MeV for light nuclei (small Z)

$U_{Coul} \sim 30$ MeV for heavier nuclei (big Z)

Should include U_{Coul} to be IPA potential energy for protons

The point is: Neutron and Proton see different effective $U(r)$

$^{120}_{50}Sn$



U_{Coul} (Coulombic, Z−1 protons)

(For protons in heavy nuclei)

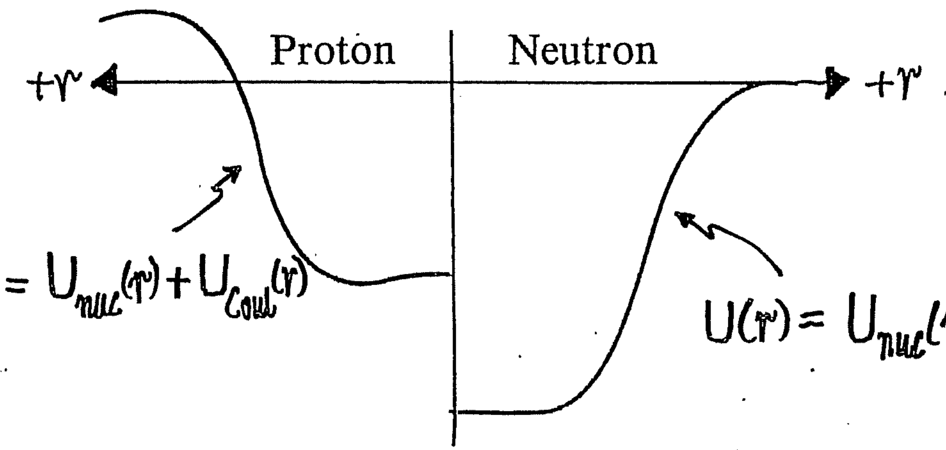
Net $U(r)$ for proton

$\boxed{U_{\text{nuc}} + U_{\text{Coul}} = U(r)\text{, for proton}}$

U_{nuc} (nuclear force, A−1 nucleons)

$\boxed{\text{Net } U(r) \text{ for neutron}}$

Different Schrödinger Equations for proton and for neutron



$U(r) = U_{nuc}(r) + U_{Coul}(r)$

$U(r) = U_{nuc}(r)$

The IPA potential energy functions for a proton and a neutron in a medium-mass nucleus.

Neutron in $U_{nuc}(r)$: 3D Spherical Symmetric Problem (Sketchy)

- $\psi_{n\ell m_\ell}(r,\theta,\phi) = R_{n\ell}(r)\, Y_{\ell m_\ell}(\theta,\phi)$ and $E_{n\ell}$ (ignore spin-orbit interaction)
- $\chi(r) = r \cdot R(r)$ satisfies $-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\chi(r) + \left[U_{nuc}(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right]\chi(r) = E\chi(r)$
  $(r>0)$ $\chi(0)=0$ (B.C.)
  - Each ℓ, there can be many eigenvalues $E_{n\ell}$
    - $\ell=0$ (s) [eigenvalues labelled as 1s, 2s, 3s, ...]
    - $\ell=1$ (p) [eigenvalues labelled as 1p, 2p, 3p, ...]
    - $\ell=2$ (d) [eigenvalues labelled as 1d, 2d, 3d, ...]

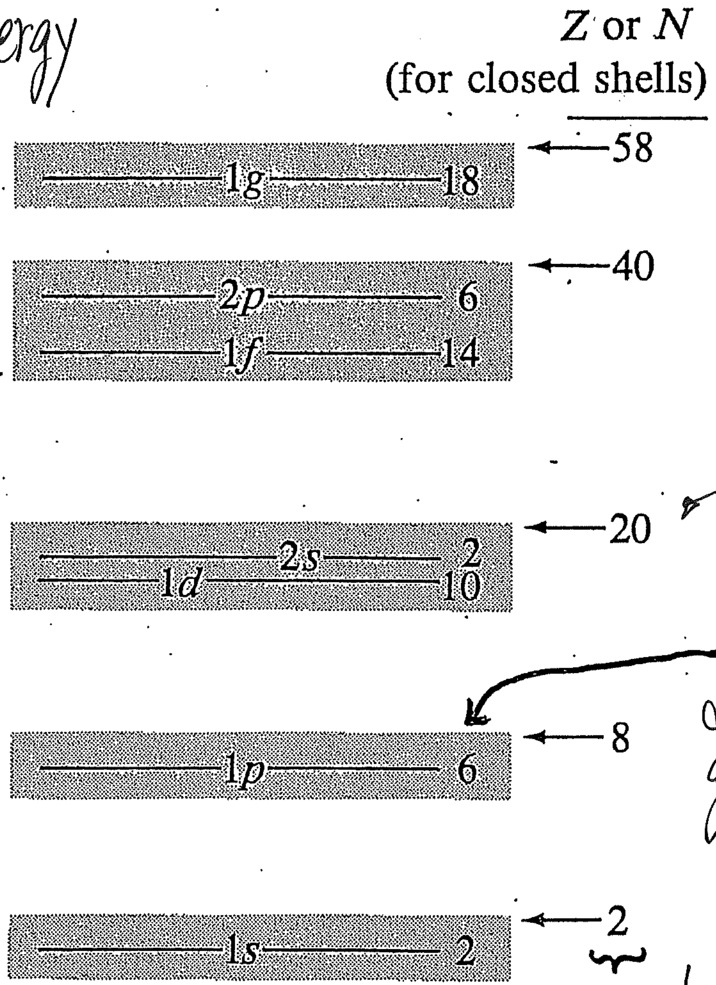
      ⋮

then line up states according to energy and fill in neutrons

[Same for proton but $U_{proton}(r) \neq U_{nuc}(r)$]

Ignore spin-orbit coupling

energy shells separated in energy



fill up to here, the next neutron/proton must go up in energy

"6" is degeneracy given by $(2l+1)\times 2$ [spin]

cummulative # of states

"Magic numbers"

Suggested more stable nuclei with Z (or N separately) being 2, 8, 20, 40, 58, ...

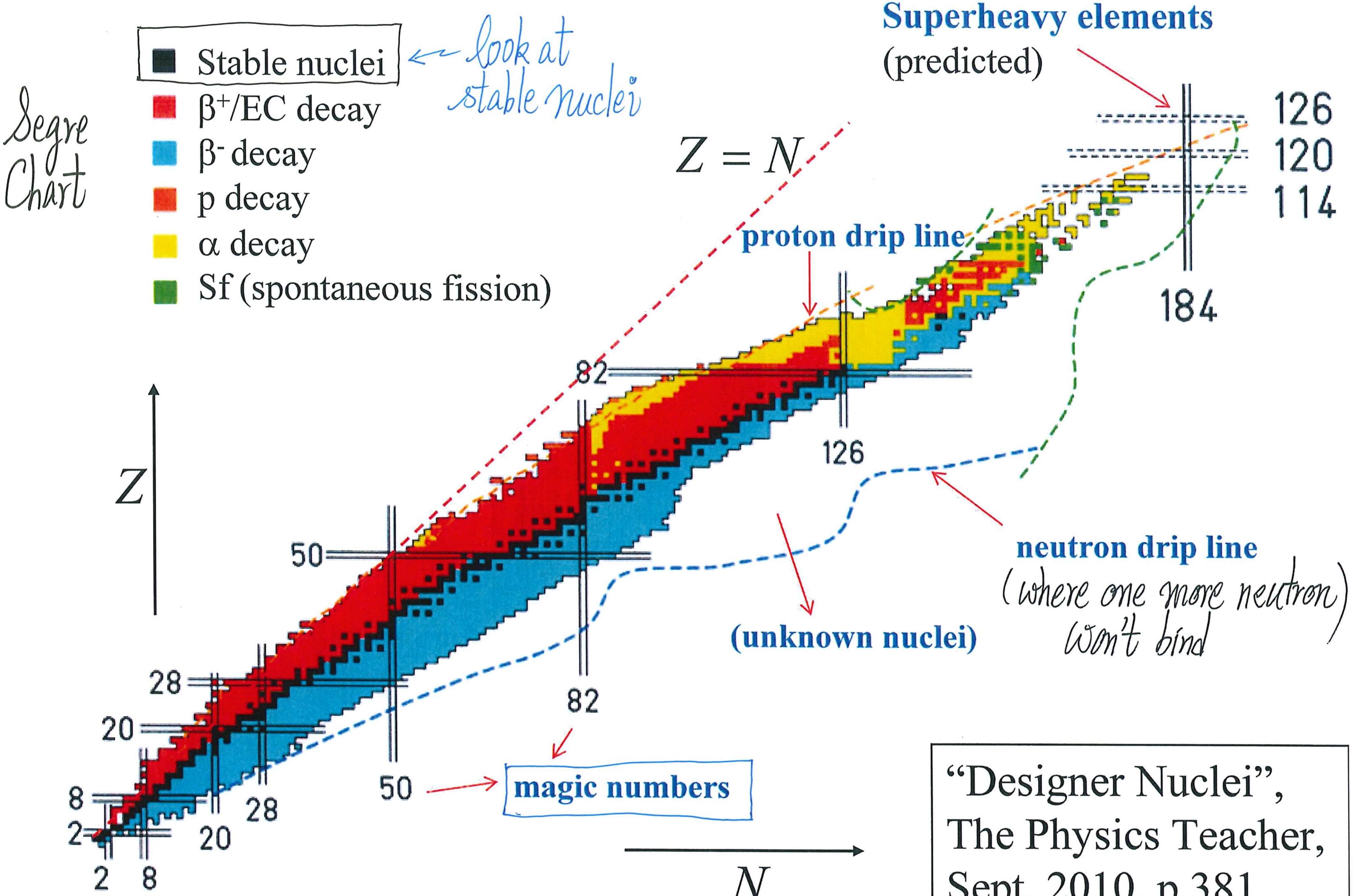
✓ ✓ ✓ $\underbrace{40, 58, \ldots}_{\text{not correct!}}$

- Energy levels for filling in protons or neutrons following Pauli Exclusion Principle

Compare with "shell structure" in atoms!

Chart of nuclei: Nuclei found in nature & lab

CC-NP-16



"Designer Nuclei", The Physics Teacher, Sept. 2010, p.381.

Experimental Data $\Rightarrow$ If Z OR N OR both take on

2, 8, (14), 20, 28, 50, 82, 126 (Magic numbers),

the nucleus is especially <u>Stable</u>.

<u>What is missing?</u> Spin-Orbit Interaction!

- $\vec{S}\cdot\vec{L}$ term will alter alignment <u>AND</u> alter degeneracy

e.g. $\ell = 1 \nearrow j = 1/2 \quad (2j+1) = 2$

$ \searrow j = 3/2 \quad (2j+1) = 4$

$\underbrace{\text{Marie G. Mayer}}_{\text{student of Born}}$ & J. Jensen (1949): included spin-orbit interaction

(Nobel Physics Prize 1963)

"Shell Model"

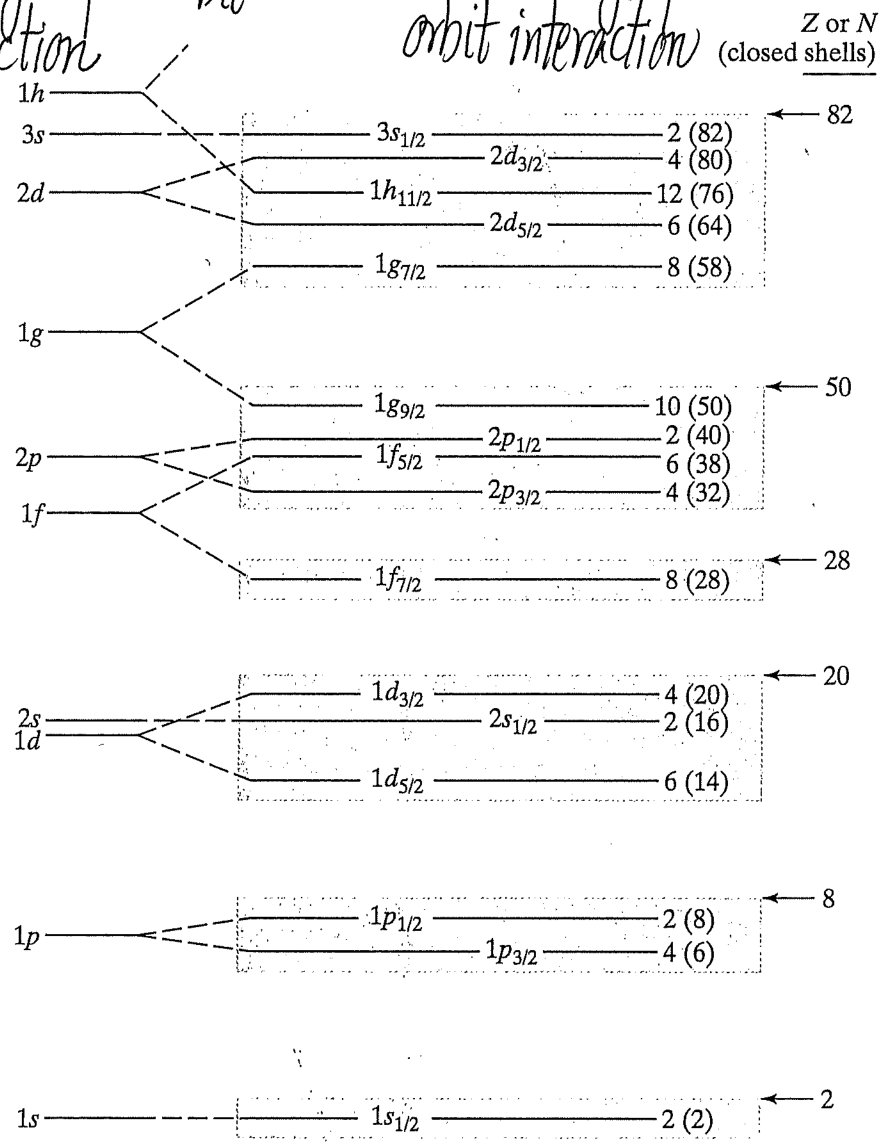
In atoms, the magic numbers are 2, 10, 18, 36, 54, 86 for closed-shell Group VIII elements.

[He Ne Ar Kr Xe Rn]

CC-NP-18

Ignoring Spin-Orbit interaction

Including spin-orbit interaction



(2j+1) cummulative #

2, 8, (14), 20, 28, 50, 82

Magic numbers in agreement with observed values

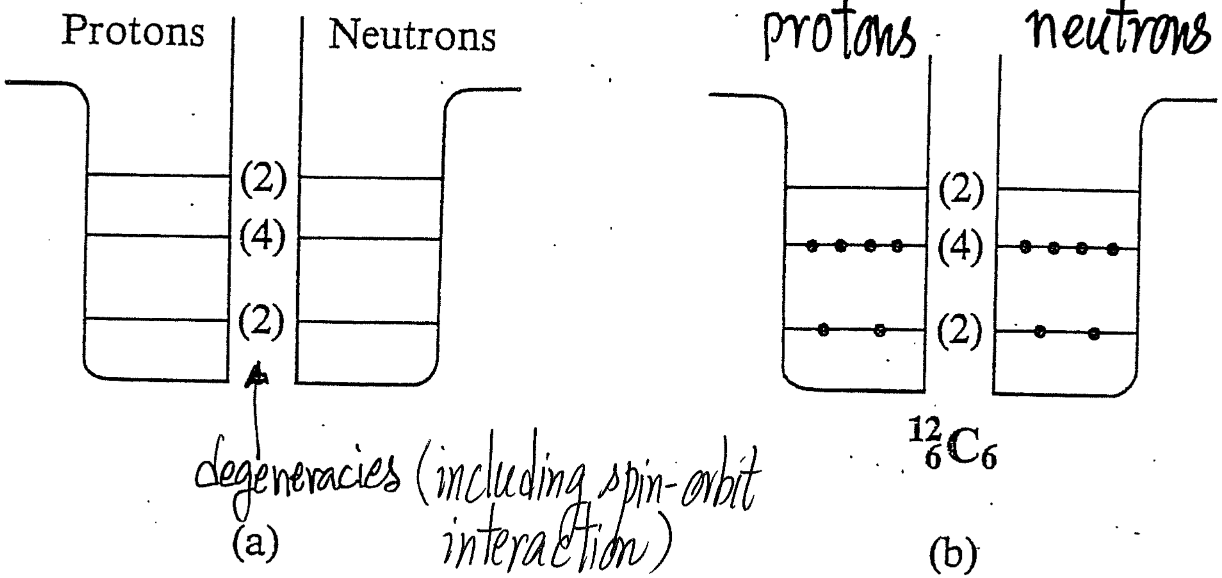
[Taken from Taylor et al. "Modern Physics for Scientists and Engineers"]

Filling order of the levels through Z or $N = 82$ for a single nucleon in a nucleus, including the spin-orbit energy proposed by Goeppert-Mayer and Jensen. The levels on the left are the corresponding levels in the absence of the spin-orbit energy. The numbers to the right of each level are the level's degeneracy and (in parentheses) the running total of protons or neutrons needed to fill through that level. On the far right are the closed-shell numbers, which agree perfectly with the observed magic numbers . The ordering of certain nearby levels is ambiguous (just as it is in atoms) and can be different for protons and neutrons. Beyond 82, where the proton well is strongly distorted by Coulomb repulsion, the level orderings for protons and neutrons are significantly different.

More detailed model later developed by A. Bohr & B. Mottelson (1975 Nobel Physics Prize)

IPA for Light Nuclei and Heavy Nuclei

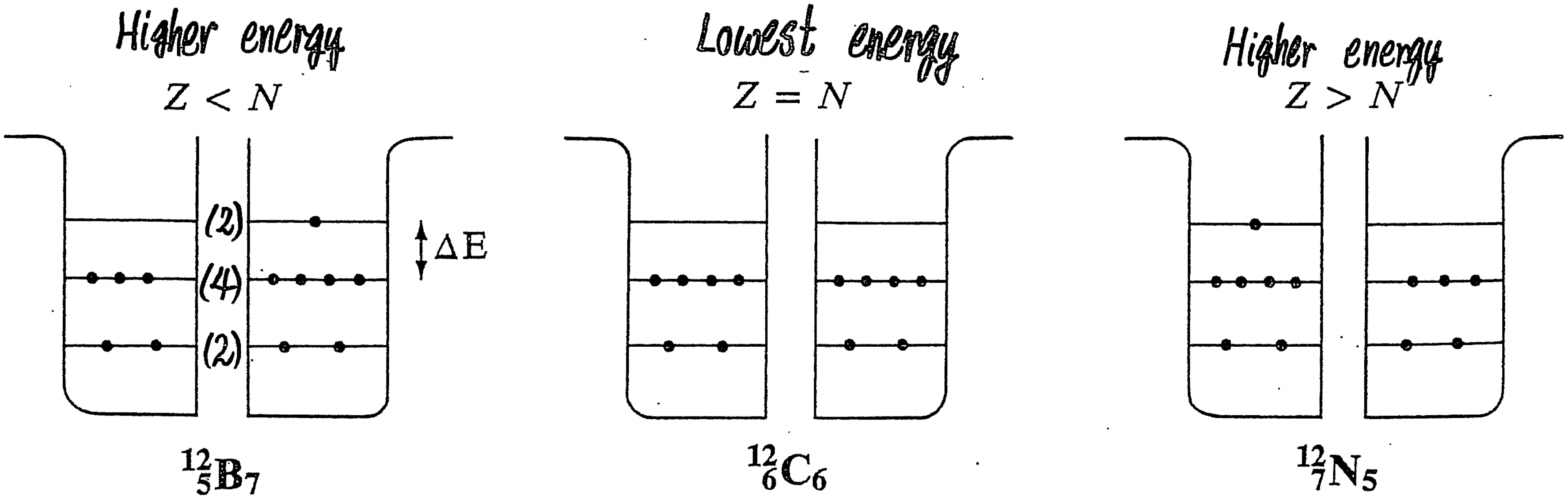
<u>Scenario 1:</u> <u>Light Nuclei</u> (Z not big) $\Rightarrow U_{proton} \approx U_{neutron}$



(a) Schematic diagram of the IPA wells and levels for protons and neutrons in a light nucleus. The two wells are essentially identical because of charge independence. The numbers in parentheses are the observed degeneracies. **(b)** The ground state of $^{12}_{6}C_6$ is found by putting its six protons in the lowest available proton levels and its six neutrons in the lowest available neutron levels. (In this case $Z=N$)

Expect for small A, $Z \approx N$ (e.g. 6 protons + 6 neutrons in ^{12}C)

Meaning: Isobars (same A), the one with $Z = N$ is most stable



The ground states of the three isobars ^{12}B, ^{12}C, and ^{12}N. Because of the Pauli principle the two nuclei with $Z \neq N$ have higher energy by the amount shown as ΔE.

<u>Radioactive decay</u>: Higher energy isobars are unstable and eventually convert into the isobar with lower energy [β-decay]

This is why $Z \approx N$ in many light stable nuclei.

$U_{proton} \approx U_{neutron}$

<u>Implication</u>: Too many neutrons ($N > Z$), if neutron can turn into proton

$\Rightarrow$ more stable

$$n \rightarrow p + e^- + \underbrace{\bar{\nu}_e}_{\text{anti-neutrino}} \quad (\beta \text{ or } \beta^- \text{ decay})$$

[we started to have radioactive decays!]

(all form basic QM + techniques in atoms/molecules)

Too many protons, if proton can turn into neutron

$$p \rightarrow n + \underbrace{e^+}_{\text{positron}} + \nu_e \quad \underline{(\beta^+ \text{ decay})}$$

OR

$$\underbrace{p + e^-}_{\text{electron capture}} \rightarrow n + \nu_e \quad (\text{electron capture})$$

[capture an electron (one closer to nucleus)]

Chart of nuclei: Nuclei found in nature & lab

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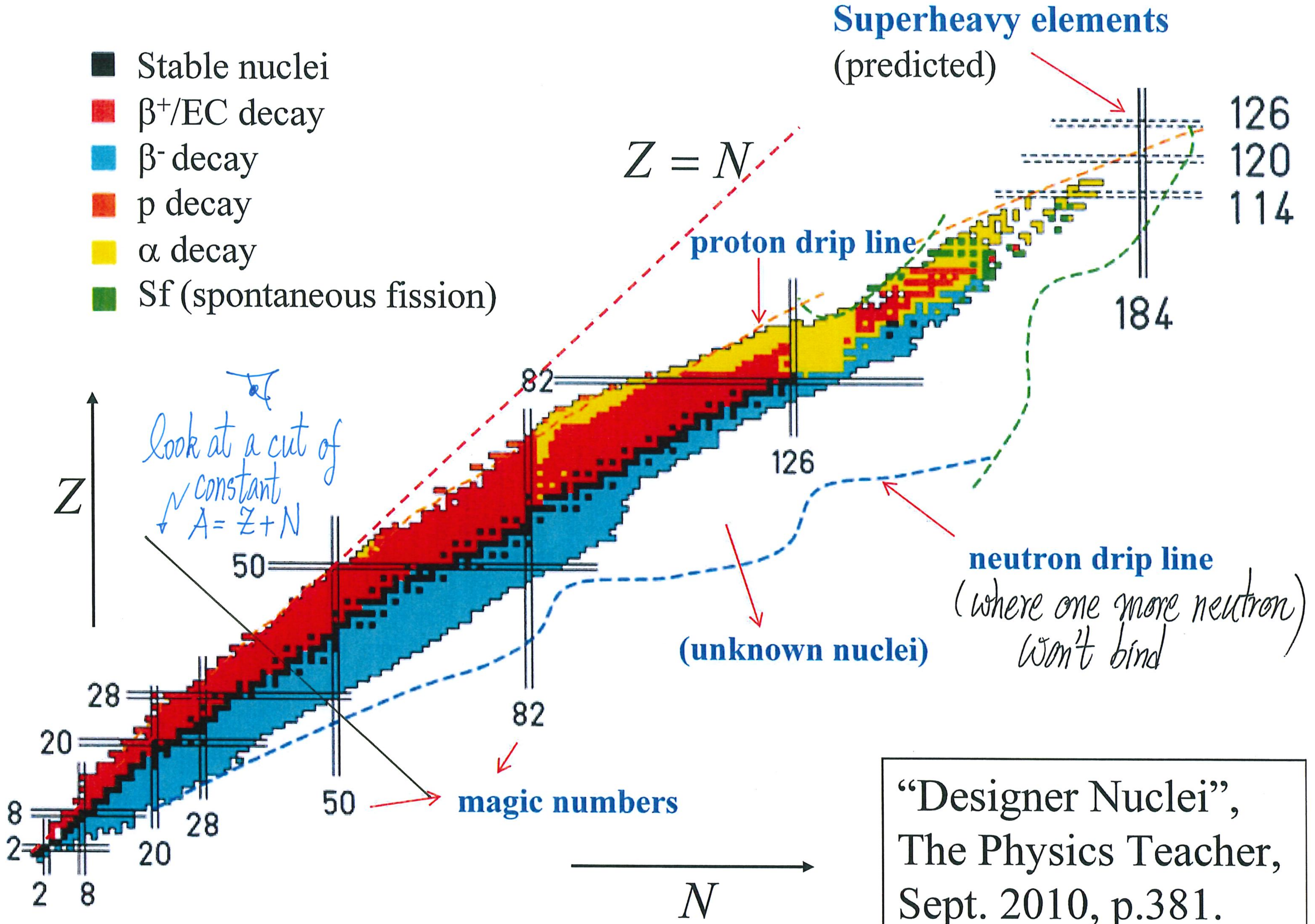
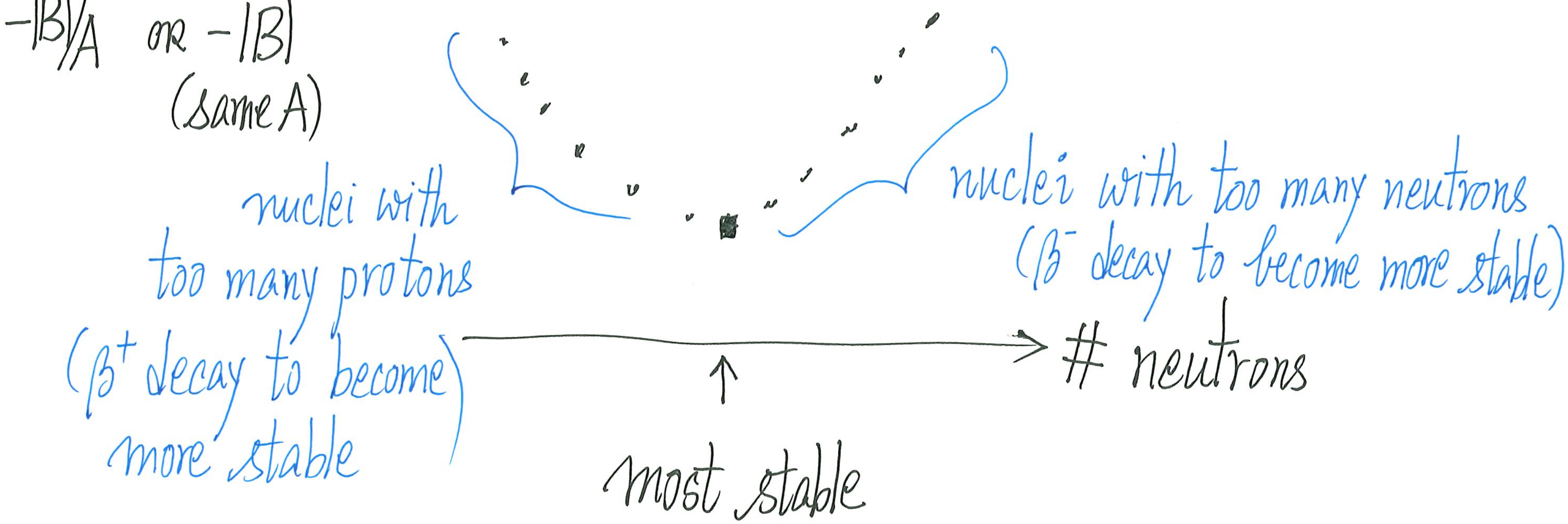


"Designer Nuclei", The Physics Teacher, Sept. 2010, p.381.

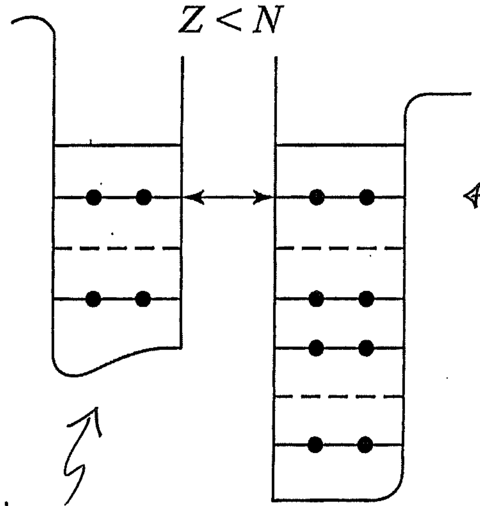
A cut of constant $A = Z + N$



Why are there radioactive decays?

Scenario 2: Heavy Stable Nuclei (Big A) $\Rightarrow U_{neutron} \neq \underbrace{U_{proton}}_{\text{shallower}}$

In nuclei with many protons the Coulomb repulsion pushes the proton well appreciably higher than the neutron well. For a given number of nucleons, the lowest energy is obtained when the highest occupied proton and neutron levels have the same energy (as indicated by the arrow). This requires that Z be somewhat less than N.

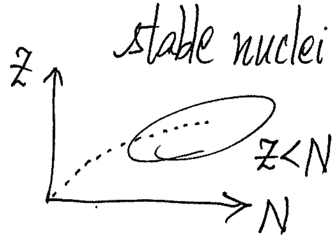


← level off

$\Rightarrow$ neutron won't tend to decay to proton, and vice versa

shallower $U_{proton}(r)$

$\Rightarrow Z < N$ for big (A) nuclei



- Simple QM, IPA, Pauli Principle, Spin-orbit Interaction predict
  - Magic numbers
  - $N \approx Z$ for light nuclei
  - $N > Z$ for heavy nuclei

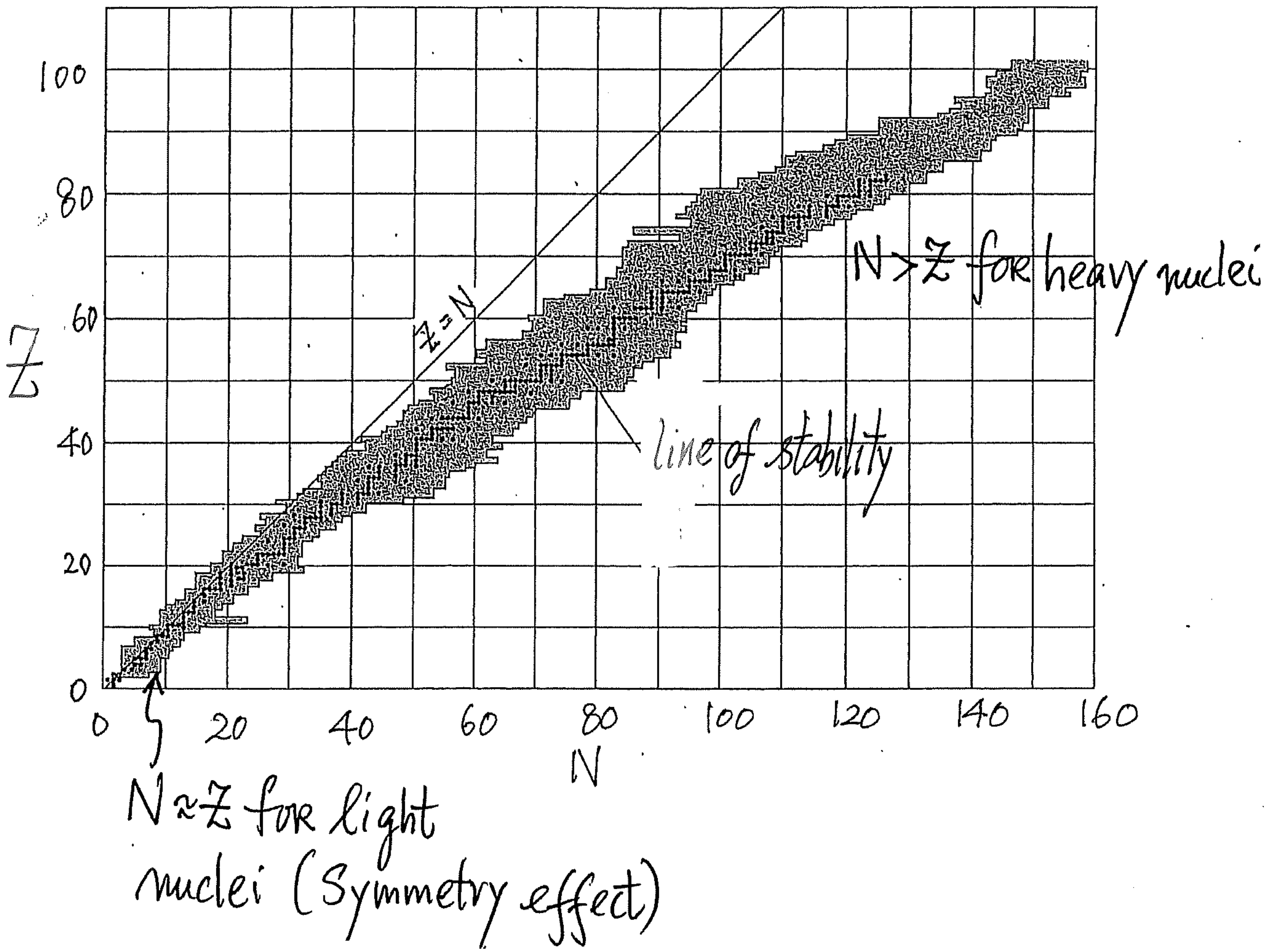
(iv) Some Properties (Facts) of Nuclei

- Naturally occuring (not man-made) — $1 \le Z \le 92$ ← Uranium; $0 \le N \le 146$
- More stable nuclei with even Z and even N

Z	N	# stable nuclei
Even	Even	148
Even	Odd	51
Odd	Even	49
Odd	Odd	4

- Magic numbers: 2, 8, 20, 28, 50, 82, 126 (for Z or N)
- Light Nuclei ($A \lesssim 40$): $N \approx Z$
- Heavy Nuclei: $N > Z$ [heaviest ones have $N \sim 1.5 Z$]

• stable nuclei



E.g.
N = 82 has several stable nuclei (with different Z)
82 is a magic number

Frontier research: Produce nuclei near boundary (highly unstable nuclei)

Chart of nuclei: Nuclei found in nature & lab

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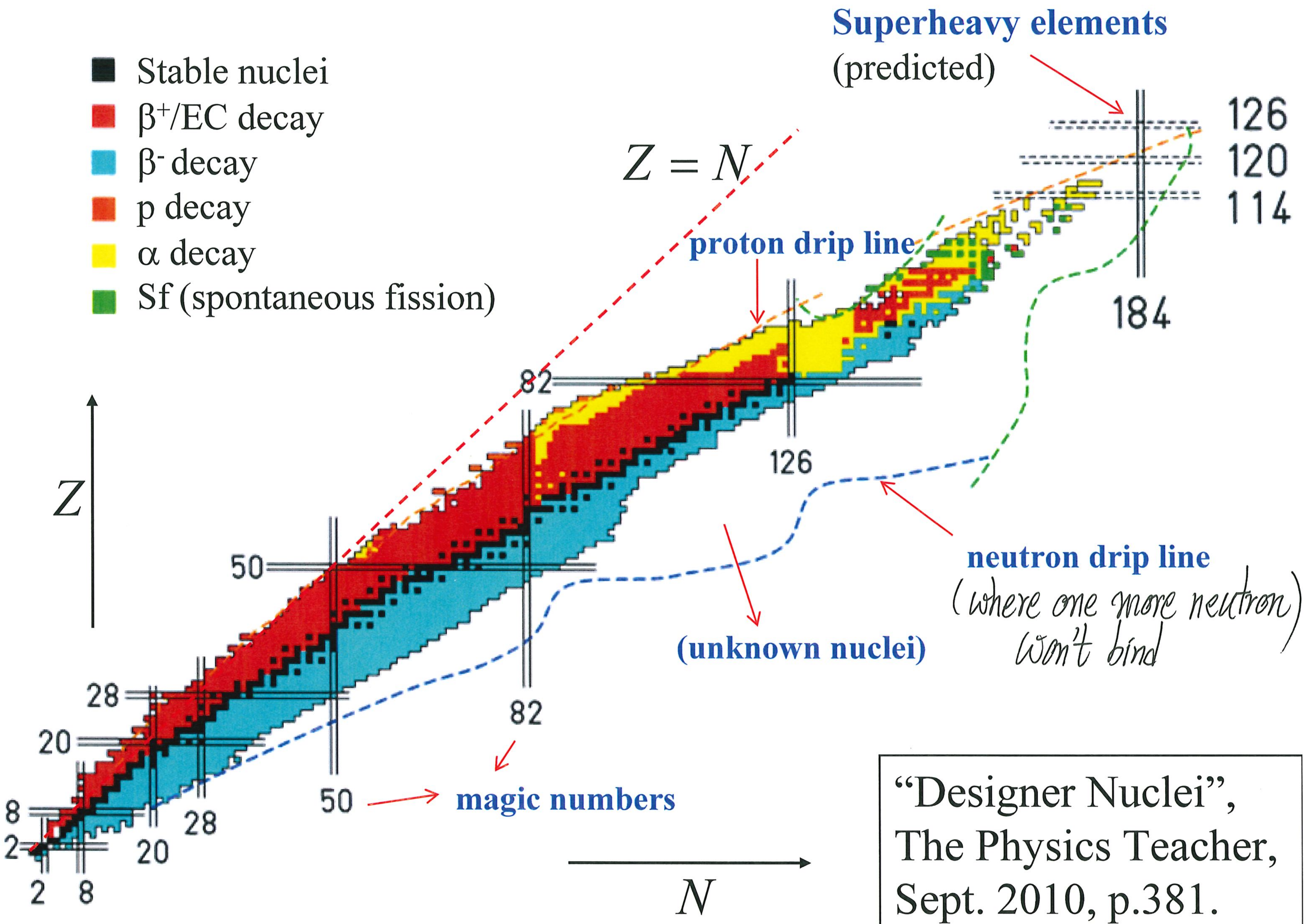


"Designer Nuclei", The Physics Teacher, Sept. 2010, p.381.

Features:

1. Stable nuclei:
   lie on a line of $Z(N)$, called **the β-stability line**.
   The line: $N \approx Z$ for small Z.
   $N \approx 1.5Z$ for large Z

2. Unstable nuclei:
   * β decay occurs for nuclei away from the β-stability line
   * decay products approaching the stability line
   * β^+/EC for nuclei in region above the line
   * β^- for nuclei in region below the line
   * larger separation from the line $\Rightarrow$ shorter $t_{1/2}$ (more unstable)
     (in white area of the chart, $t_{1/2}$ is too short to be measured)
   * α decay: for $A > 140$
   * fission: for large A

Unstable Nuclei could undergo:

α-decay: $\,^{A}_{Z}X \rightarrow \,^{A-4}_{Z-2}Y + \,^{4}_{2}He$ (e.g. $\,^{238}_{92}U \rightarrow \,^{234}_{90}Th + \,^{4}_{2}He$)

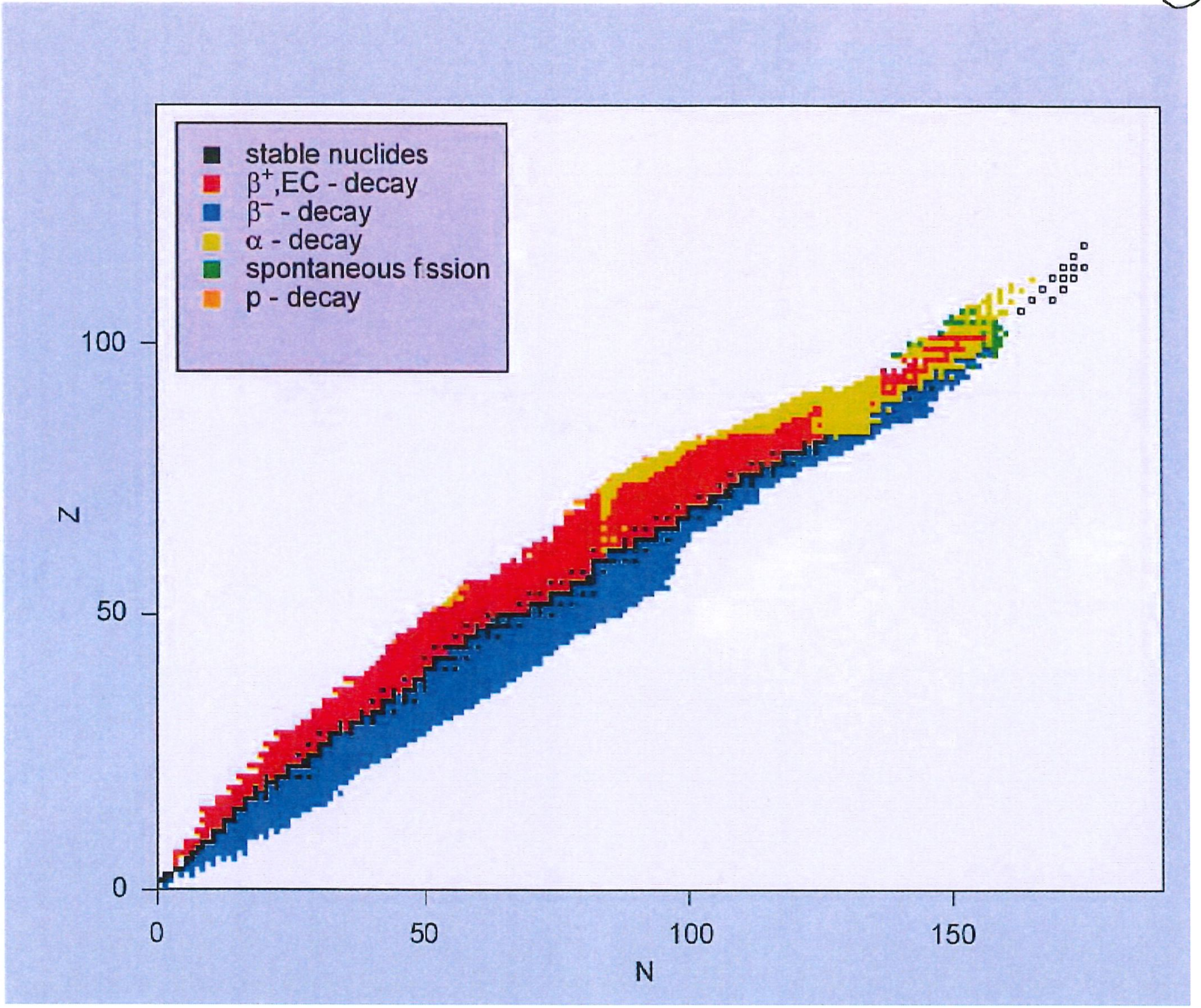
β-decay: $\,^{A}_{Z}X \rightarrow \,^{A}_{Z+1}Y + e^{-} + \bar{\nu}_e$ $(n \rightarrow p + e^{-} + \bar{\nu}_e)$

β^{+}-decay: $\,^{A}_{Z+1}Y \rightarrow \,^{A}_{Z}X + e^{+} + \nu_e$

Electron Capture (EC): $\,^{A}_{Z+1}Y + e^{-} \rightarrow \,^{A}_{Z}X + \nu_e$

Segrè chart

The presently known nuclei displayed according to proton number Z on the vertical scale, and neutron number N on the horizontal scale. The black squares denote the stable nuclei, and also the extremely long lived nuclei like ^{238}U that exist on Earth. The blue squares indicate nuclei with an excess of neutrons and so beta decay by emitting electrons. The red squares indicate nuclei which undergo positron decay (or electron capture). The yellow nuclei are those which decay by emitting alpha particles and the green nuclei undergo spontaneous fission. There are a few orange nuclei along the upper edge of the coloured area; these decay by emitting protons. The squares at the top right are recently produced super-heavy nuclei. This sort of diagram is called a Segrè chart.



This is an important chart in nuclear physics. Besides the stable nuclei (black squares), many other unstable nuclei exist and they undergo various decay processes (here represented by different colors). [Taken from "Nucleus" by MAJP.]

"Nucleus: A trip into the heart of Matter" by Mackintosh, Al-Khalili, Jonson, Peña

(v) Activity, Lifetime, half-life

- "Activity" quantifies the "rate" at which decays occur for a sample
- $N(t)$ = # radioactive nuclei in sample at time t (c.f. # atoms $N_2(t)$ in excited state at time t)

$$\boxed{\text{Activity}^{+}\ R = -\frac{dN(t)}{dt}}$$

- R is positive $\left(\frac{dN}{dt} < 0\right)$
- Unit: 1/time (s^{-1}) (c.f. transition rate)

1 decay/second = 1 s^{-1} = 1 Becquerel = 1 Bq

More commonly, MBq = 10^6 Bq or GBq = 10^9 Bq

SI units

Old Unit: 1 Curie = 37 GBq

Experiments $\Rightarrow$ $R \sim e^{-\lambda t}$ $\left(\frac{1}{\lambda} = \tau = \text{life time}\right)$

+ Unfortunately, R is also the symbol for Activity. It shouldn't be confused with range R and radius R

Implication: Radioactivity is <u>statistical</u> in nature

λ = <u>Prob.</u> for <u>a nucleus</u> in sample to decay <u>per unit time</u> (1/time)

$$R(t) = \#\text{ decays per unit time} = \lambda \cdot N(t) = -\frac{dN(t)}{dt} \quad (+)$$

$$\Rightarrow N(t) = \underbrace{N_0}_{N(0)\text{ a constant}} e^{-\lambda t} \Rightarrow R(t) = \underbrace{\lambda N_0}_{R(0)} e^{-\lambda t} \sim e^{-\lambda t} \text{ as observed}$$

$$\frac{1}{\lambda} \equiv \tau = \underline{\text{Lifetime}}^{+} \qquad {}^{226}_{88}Ra \text{ has } \lambda = 1.4 \times 10^{-11}\,s^{-1} \Rightarrow \tau = 7.14 \times 10^{10}\,s$$

<u>Half-life $t_{1/2}$</u>: $N(t_{1/2}) = \dfrac{N_0}{2}$ defines $t_{1/2}$

$$\boxed{t_{1/2} = (\ln 2) \cdot \tau = 0.693\,\tau} \qquad {}^{226}_{88}Ra \quad t_{1/2} = 5 \times 10^{10}\,s$$

$^{+}$Note: This is what we did for the spontaneous emission in atoms.

(vi) Q-value (Dis-integration energy) in α-decay

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Atomic masses M_X, M_Y ... M_α

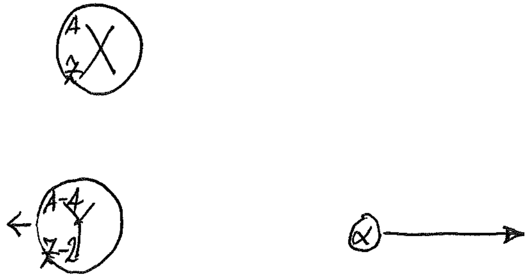
$$\boxed{Q = (M_X - M_Y - M_\alpha)\cdot c^2 = (\Delta M)c^2}$$

- energy equivalent of mass difference
- If X is at rest and decays, Q is available to Y and α as k.e.'s

At rest

Before

After



$M_\alpha \ll M_Y$

$\Rightarrow$ $\boxed{\alpha\text{-particle carries most of } Q \text{ as its k.e.}}$

$$\boxed{K_\alpha = \frac{(\Delta M)c^2}{1 + \frac{M_\alpha}{M_Y}} = \underbrace{\frac{A-4}{A}}_{\text{large part of } Q}\cdot Q}$$

K_α: k.e. of α-particle

- followed from conservation of energy and conservation of momentum

At this point, we go back to the <u>application of tunneling</u> as the mechanism of α-decay in big nuclei